Quantum Decoherence in Disordered Mesoscopic Systems

Dmitrii S. Golubev[1,2] and Andrei D. Zaikin[2,3]
[1] *Physics Department, Chalmers University of Technology, S-41296 Göteborg, Sweden*
[2] *I.E.Tamm Department of Theoretical Physics, P.N.Lebedev Physics Institute, Leninskii pr. 53, 117924 Moscow, Russia*
[3] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany*

We point out that the low temperature saturation of the electron phase decoherence time in a disordered conductor can be explained within the existing theory of weak localization provided the effect of quantum (high frequency) fluctuations is taken into account. Making use of the fluctuation-dissipation theorem we evaluate the quantum decoherence time, the crossover temperature below which thermal effects become unimportant, and the weak localization correction $\delta\sigma$ at $T = 0$. For 1d systems the latter is found to be $\delta\sigma \propto 1/\sqrt{N}$, where N is the number of conducting channels.

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Quantum interference between electrons has a strong impact on electron transport in a disordered metal, leading to the so-called weak localization correction to the system conductance [1]. This correction is large provided the electrons moving in the metal remain coherent. On the other hand, this phase coherence can persist only for a finite time and is eventually destroyed due to various processes, such as electron-electron and electron-phonon interactions, spin-flip scattering, etc. This characteristic decoherence time τ_φ plays a prominent role in the theory of weak localization [1,2].

In the absence of magnetic impurities and if the temperature of the system is sufficiently low the decoherence time τ_φ is determined by electron-electron interactions. It was demonstrated in Ref. [3] (see also [2,4]) that for this dephasing mechanism the decoherence time increases with temperature as $\tau_\varphi \propto T^{2/(d-4)}$, where d is the system dimension. This theoretical prediction was verified in several experiments [5,6] over a certain temperature interval.

Does the divergence of τ_φ in the zero temperature limit imply that coherence is not destroyed at $T = 0$? Recent experiments [7] clearly suggest a negative answer, indicating that at very low temperatures the time τ_φ saturates at a finite level showing no tendency for further increase with decreasing T. The authors [7] argued that this saturation is not caused by heating or magnetic impurities but rather is a fundamental consequence of zero-point fluctuations of electrons. A saturation of τ_φ at low T was also observed in earlier works (see e.g. [5,6]).

The aim of this paper is to demonstrate that the observed saturation of τ_φ at lowest temperatures [7] can be explained within the existing theory of weak localization [2] if one takes into account quantum fluctuations of the electric field in a disordered conductor.

We essentially follow the analysis elaborated by Chakravarty and Schmid [2] and consider the propagation of an electron with the kinetic energy $m\dot{\boldsymbol{r}}^2/2$ in a potential of randomly distributed impurities $U_{imp}(\boldsymbol{r})$. In addition to that the electron interacts with the fluctuating electric field $\boldsymbol{E}(\boldsymbol{r}, t) = -\nabla V(\boldsymbol{r}, t)$ produced by other electrons. These electrons play the role of an effective environment.

Let us express the propagating electron amplitude in terms of the Feynman path integral. Within the quasi-classical approximation (which is sufficient as long as the elastic mean free path l exceeds the Fermi wavelength $p_F l \gg 1$) the path integral can be replaced by the sum over the classical trajectories obeying the equation of motion

$$m\ddot{\boldsymbol{r}} = -\nabla U_{imp}(\boldsymbol{r}) - e\nabla V(\boldsymbol{r}, t) \tag{1}$$

for each realization of random potentials $U_{imp}(\boldsymbol{r})$ and $V(\boldsymbol{r}, t)$. Averaging over disordered configurations of impurities [2] yields the effective picture of electron diffusion at the scales bigger than l. Fluctuations of the electric field $\nabla V(\boldsymbol{r}, t)$ lead to the phase decoherence. Defining the phase difference between a classical electron path $\boldsymbol{r}(t')$ and a time reversed path $\boldsymbol{r}(t - t')$

$$\delta\varphi(\boldsymbol{r}, t) = -e\int_0^t dt'[V(\boldsymbol{r}(t'), t') - V(\boldsymbol{r}(t - t'), t')] \tag{2}$$

(which is nonzero provided V fluctuates in space and time) and averaging with respect to fluctuations of V, for not very small t one gets [2]

$$\langle(\delta\varphi(\boldsymbol{r}, t))^2\rangle/2 = t/\tau_\varphi(T), \tag{3}$$

where

$$\frac{1}{\tau_\varphi(T)} = \frac{e^2}{a^{3-d}}\int dt\int\frac{d\omega d^dq}{(2\pi)^{d+1}}\langle|V_{q,\omega}|^2\rangle e^{-Dq^2|t|-i\omega t}, \tag{4}$$

a is the film thickness for $d = 2$ and $a^2 = s$ is the wire cross section for $d = 1$.

The correlation function for voltages in (4) can be determined with the aid of the fluctuation-dissipation theorem [8]. For the sake of definiteness let us consider a quasi-one-dimensional conductor. Then one finds

$$\langle|V_{q,\omega}|^2\rangle = \frac{\omega\coth\left(\frac{\omega}{2T}\right)}{\frac{\omega^2C^2}{\sigma q^2} + \sigma q^2(1 + \frac{CD}{\sigma})^2}. \tag{5}$$

Here $\sigma = 2e^2N_0Ds$ is the classical Drude conductance, D is the diffusion coefficient, and C is the capacitance of a linear conductor per unit length. In (5) we neglected retardation and skin effects which may become important only at very high frequencies. Substituting (5) into (4) and integrating over t and q after a trivial algebra we find

$$\frac{1}{\tau_\varphi(T)} = \frac{e^2\sqrt{2D}}{\sigma}\int\limits_{1/\tau_\varphi}^{1/\tau_e}\frac{d\omega}{2\pi}\frac{\coth(\omega/2T)}{\sqrt{\omega}}. \tag{6}$$

In eq. (6) we made use of the condition $C \ll \sigma/D$ which is usually well satisfied (perhaps except for extremely thin wires) indicating the smallness of capacitive effects in our system. Eq. (6) yields

$$\frac{1}{\tau_\varphi} = \frac{e^2}{\pi\sigma}\sqrt{\frac{2D}{\tau_e}}\left[2T\sqrt{\tau_e\tau_\varphi}+1\right]. \tag{7}$$

The first term in the square brackets comes from the low frequency modes $\omega < T$ whereas the second term is due to high frequency ($\omega > T$) fluctuations of the electric field in a disordered conductor. At sufficiently high temperature the first term dominates and the usual expression [3] $\tau_\varphi \sim (\sigma/e^2D^{1/2}T)^{2/3}$ is recovered. As T is lowered the number of the low frequency modes decreases and eventually vanishes in the limit $T \to 0$. At $T \lesssim T_q \sim 1/\sqrt{\tau_\varphi\tau_e}$ the expression (7) is dominated by the second term and τ_φ saturates at the value

$$\tau_\varphi \approx \pi\sigma/e^2v_F \tag{8}$$

(we disregard the numerical prefactor of order one). The estimate for the crossover temperature T_q reads

$$T_q \sim ev_F/\sqrt{\sigma l}. \tag{9}$$

Making use of eq. (8) it is also easy to find the weak localization correction $\delta\sigma$ to the Drude conductance in the limit $T = 0$. For $T \lesssim T_q$ we obtain

$$\frac{\delta\sigma}{\sigma} = -\frac{e^2}{\pi\sigma}\sqrt{D\tau_\varphi} \approx -\frac{1}{p_Fs^{1/2}}, \tag{10}$$

i.e. $\delta\sigma \approx -\sigma/\sqrt{N}$, where $N \sim p_F^2s$ is the effective number of conducting channels in a 1d mesoscopic system.

For 2d and 3d systems the same analysis yields

$$\frac{1}{\tau_\varphi} = \frac{e^2}{4\pi\sigma\tau_e}[1+2T\tau_e\ln(T\tau_\varphi)], \qquad \text{2d},$$
$$\frac{1}{\tau_\varphi} = \frac{e^2}{3\pi^2\sigma\sqrt{2D}\tau_e^{3/2}}[1+6(T\tau_e)^{3/2}], \quad \text{3d}, \tag{11}$$

where $\sigma = 2e^2N_0Da^{3-d}$ is the conductance of a d-dimensional system. The result (11) demonstrates that for 2d and 3d systems saturation of τ_φ is expected already at relavitely high temperatures: the corresponding crossover temperature T_q is of the order of the inverse elastic time in the 3d case and $T_q \sim v_F/l\ln(p_F^2al)^2$ for a 2d system. The latter value agrees well with the experimental results [5].

The physical origin of the decoherence time saturation at low temperatures is quite transparent: in the limit $T \to 0$ the dephasing effect is due to quantum fluctuations of the electric field produced by electrons in a disordered conductor. This decoherence effect is by no means surprizing. In fact, it is well known that even at $T = 0$ interaction of a quantum particle with an external quantum bath leads to the loss of quantum coherence and – under certain conditions – to localization of this particle (see e.g. [9,10]).

Our analysis clearly suggests that at sufficiently low temperatures the decoherence time τ_φ is *not* equal to the inelastic mean free time τ_i, which is known to become infinite at zero temperature for almost all processes, including electron-electron interaction. In order to find τ_i it is sufficient to proceed within the standard quasiclassical approach and to solve the kinetic equation for the electron distribution function. The collision integral in this equation contains the product of the occupation numbers for different energy levels $n_k(1-n_q)$, which vanishes at $T \to 0$ due to the Pauli principle. As a result τ_i diverges in the zero temperature limit.

In terms of the path integral analysis this procedure amounts to expanding the electron effective action on the Keldysh contour in the parameter $\boldsymbol{r}_-(t') = \boldsymbol{r}_1(t') - \boldsymbol{r}_2(t')$ assuming this parameter to be small ($\boldsymbol{r}_{1(2)}(t')$ is the electron coordinate on the forward (backward) part of the Keldysh contour). This procedure is formally very different from one used to calculate the weak localization correction to conductivity [2]. In the latter case time reversed pathes $\boldsymbol{r}_1(t')$ and $\boldsymbol{r}_2(t-t')$ are assumed to be close to each other whereas $\boldsymbol{r}_-(t')$ can be arbitrarily large. This formal difference is just an illustration of the well know fact, that weak localization is an essentially quantum phenomenon. Therefore, the standard quasiclassical kinetic analysis of τ_i in terms of the collision integral – especially at the lowest temperatures – appears to be insuffient for calculation of the decoherence time.

It is also interesting to point out that the expression for the electron-electron inelastic time τ_i^{ee} (see e.g. [1]) is determined by the integral which (apart from an unimportant numerical prefactor) coincides with the high frequency part ($\omega > T$) of the integral (4,5). In the case of τ_i^{ee} the integral has the high frequency cutoff at the electron energy $\epsilon \sim T$, and one obtains [1] $1/\tau^{ee} \propto \epsilon^{d/2} \propto T^{d/2}$. Comparing this expression for $1/\tau_i^{ee}$ with our results for the inverse decoherence time $1/\tau_\varphi$ we arrive at the conclusion that the former is *never* important as compared to the latter: at high $T > T_q$ the inverse decoherence time is determined by the low frequency Nyqist noise $\omega \ll T$, whereas at low $T < T_q$ the

main contribution to $1/\tau_\varphi$ comes from the high frequency modes of the electric field fluctuations $\omega \gg T$. In both cases we have $1/\tau_\varphi \gg 1/\tau_i^{ee}$.

We would like to emphasize that our results are obtained within the standard theoretical treatment of weak localization effects [2] combined with the fluctuation-dissipation theorem. One can elaborate a more general analysis starting from the microscopic Hamiltonian for electrons in a disordered metal with Coulomb interaction, introducing the quantum field V by means of a Hubbard-Stratonovich transformation (see e.g. [10]) and deriving the effective action for one electron after integration over the remaining electron degrees of freedom which play the role of the bath. In the quasiclassical limit $p_F l \gg 1$ one arrives at the same results as those obtained here.

Note that the decoherence time saturation at low T has been also discussed in a very recent preprint by Vavilov and Ambegaokar [11]. These authors describe the dephasing effect of electromagnetic fluctuations by means of the effective Caldeira-Leggett bath of oscillators coupled to the electron coordinate. As compared to our treatment, there are at least two important differences: (i) the model [11] does not account for spacial fluctuations of the electromagnetic field in the sample and (ii) even at lowest temperatures the authors [11] treated fluctuations of the bath as a white noise with temperature T (cf. eq. (11) of Ref. [11]). Within this model saturation of the decoherence time at $T = 0$ was obtained only due to the finite sample size: the corresponding value τ_φ [11] tends to infinity as the sample length becomes large. In contrast, our results (6-10) do not depend on the length of the conductor.

Our result for the quantum decoherence time (8) also appears to be different from that presented by Mohanty, Jariwala and Webb (eq. (2) of Ref. [7]). Note, however, that numerical values for τ_φ obtained from our eq. (8) for the samples Au-1,3,4,6 of [7] are in a surprizingly good agreement with the corresponding estimates derived in Ref. [7]. The latter in turn agree with the experimental data obtained in [7].

Weak localization corrections to the conductance of 1d wires have been also investigated by Pooke *et al.* [6]. At very low temperatures these authors observed a finite length $L_\varphi = \sqrt{D\tau_\varphi}$, which scales as $\sqrt{\sigma}$ (with other parameters being fixed) in agreement with our eq. (8).

In 2d films the decoherence time saturation at low T was experimentally found in Ref. [5]. The authors attributed this effect to spin-spin scattering. In our opinion (which seems to be shared by the authors [5]) this explanation is not quite satisfactory because it does not allow to understand the linear dependence of $1/\tau_\varphi$ on the sheet resistance of the film detected in [5]. In contrast, this dependence can be easily explained within the analysis developed here. The result (11) is in a quantitative agreement with the experimental findings [5].

In conclusion, we point out that the low temperature saturation of the electron decoherence time found in recent experiments with mesoscopic conductors can be explained within the existing theory of weak localization provided the effect of intrinsic quantum fluctuations of the electric field is properly accounted for. Our results agree well with the experimental data.

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